COORDINATE SYSTEM FORMULATIONS FOR INTEGRATED ATMOSPHERE–WAVE–ICE–OCEAN MODELLING

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Abstract

The effect of ocean waves on near-surface currents is important for many applications, including prediction of the drift of oil spills and other material, coastal circulation, upwelling/downwelling in the marginal ice zone, and the generation of Langmuir circulations and related phenomena. In order to predict or simulate such processes on an operational basis it is advantageous to run a coupled model system for surface waves, the ocean circulation, and the atmospheric boundary layer. Such a coupled system should take account of the following: the Earth's rotation, oceanic and atmospheric stratification, turbulence, the momentum balance during wave generation and dissipation, the effect of horizontal and vertical mean shear on wave propagation, the effect of waves on the mean water level, and the presence of sea ice and viscoelastic surface films.

An outline is given of equations which may be used for such a coupled model system, valid to second order in wave slope. A discussion is given of the advantages and disadvantages of using various different coordinate systems: Eulerian, curvilinear surface-following, Lagrangian, and generalized Lagrangian mean (GLM). Coordinate systems which follow the wave profile are able to resolve fine-scale vertical structure near the water surface. The GLM formulation provides an elegant treatment of the dynamics, but breaks down at critical levels, where the mean velocity is equal to the wave phase speed.

Introduction

Within operational oceanography, there are phenomena in which the coupled effects of ocean waves and near-surface currents are important. These include the following:

- *The drift of oil spills and other material:* Depends on the near-surface drift current profile, the amount of downmixing by surface waves, etc.
- *Coastal circulation:* Momentum from waves is transferred to the water column on wave breaking, leading to 'wave setup' and longshore drift.
- *Upwelling/downwelling in the presence of sea ice:* Sea ice radically alters the surface boundary conditions: changes in the surface boundary current will cause upwelling/downwelling via the continuity equation.
- *The generation of Langmuir circulations and related phenomena:* Resulting from longitudinal circulations due to crossing wave trains, instability of the drift current profile, distortion of vortex tubes etc.

On an operational basis, in order to predict or simulate such processes, there are clear advantages in running a coupled model system, for:

- *Surface waves;*
- *The ocean circulation;* and
- *The atmosphere, including the boundary layer above the sea surface:*

A coupled model system such as one outlined above should take account of the following:

- *The Earth's rotation:* Coriolis effect, on the total (Lagrangian mean) current;
- *Oceanic and atmospheric stratification;*
- *Turbulence;*
- *The momentum balance during wave generation and dissipation;*
- *The effect of horizontal and vertical mean shear on wave propagation:*
- *The effect of waves on the mean water level;*
- *The presence of sea ice:* Even a thin layer of grease/frazil ice or slush will radically change the surface boundary conditions, in a way which is mathematically similar to the effect of a viscoelastic surface film (see below); and
- *The presence of viscoelastic surface films:* These cause a large increase in viscous wave damping, and radically change the behaviour of the mean flow in the surface vorticity layer.

To include the effects of water waves in a coupled model for the atmospheric and oceanic boundary layers, one can either resolve individual waves, a procedure which is usually computationally uneconomic, or to use an averaging procedure. The large variations in atmospheric and oceanic properties near the air–water interface, over small vertical distances, may not be properly resolved if the averaging, is performed for fixed vertical co-ordinates (x_3). Much better resolution will be obtained if a coordinate system is used in which the interface follows a coordinate surface during the wave cycle.

Coordinate Systems

A large variety of surface-following coordinate systems is available. One may use, for example, fixed curvilinear coordinates in a a moving reference frame [17], or a Lagrangian formulation, in which the fluid particles have fixed coordinate labels [5, 23]. If we specify that the mean fluid velocity at a particular coordinate location is equal to the mean drift velocity of a fluid particle passing through the location, we have the generalized Lagrangian mean (GLM) formulation of Andrews and McIntyre [2].

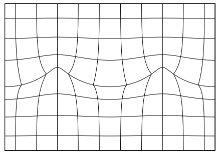

Figure 1. Example of a coordinate system, above and below a wave surface. In this particular case we have $\mathbf{x} = \left(y_1 - ae^{-k|y_3|}\sin(ky_1 - \omega t),\ y_2,\ ae^{-k|y_3|}\cos(ky_1 - \omega t)\right)$. Above the interface, the coordinate system is isomorphic: below the interface $J = 1 + O\left[(ak)^2\right]$.

General Formulation

It is advantageous to write the hydrodynamic equations in conservation-law form [1]. A general treatment which can be used for many coordinate systems is described by Jenkins [13, 14], using similar notation to Andrews and McIntyre [2]. The fixed Cartesian coordinate system is denoted by $\mathbf{x} = (x_1, x_2, x_3)$, and the curvilinear system by $\mathbf{y} = (y_1, y_2, y_3)$. Superscripts $(\cdot)^{\mathbf{x}}$ and $(\cdot)^{\mathbf{y}}$ specify the coordinate system. In the following, J represents the determinant of the Jacobian matrix $[x_{j,l}]$, and K_{jl} its cofactors. We assume the following momentum and continuity equations:

$$\rho^{\mathbf{x}}\left[u^{\mathbf{x}}_{j,t} + u^{\mathbf{x}}_l u^{\mathbf{x}}_{j,l} + \Phi^{\mathbf{x}}_{,j} + 2(\Omega \times \mathbf{u}^{\mathbf{x}})_j\right] - \tau^{\mathbf{x}}_{jl,l} = 0, \qquad \rho^{\mathbf{x}}_{,t} + u^{\mathbf{x}}_l \rho^{\mathbf{x}}_{,l} + \rho^{\mathbf{x}} u^{\mathbf{x}}_{l,l} = 0, \tag{1}$$

where ρ is the fluid density, $\mathbf{u} = (u_1, u_2, u_3)$ is the velocity, Ω is the rotational angular velocity vector, Φ is a force (e.g. gravitational) potential and the tensor τ incorporates both pressure and shear stress. Repeated indices are summed from 1 to 3.

The momentum equation (Eq. 1) becomes:

$$P_{j,t} - T_{jl,l} = S_j, \tag{2}$$

where $P_j = \rho^{\mathbf{y}} J u^{\mathbf{y}}_j$ is the 'concentration of x_j-momentum in $\mathbf{y}$-space', $T_{jl} = \left[\tau^{\mathbf{y}}_{jm} - \rho^{\mathbf{y}} u^{\mathbf{y}}_j\left(u^{\mathbf{y}}_m - x^{\mathbf{y}}_{m,t}\right)\right] K_{ml}$ is minus the flux of x_j-momentum across y_l-surfaces, and $S_j = -\rho^{\mathbf{y}}\Phi^{\mathbf{y}}_{,l} K_{jl} - 2\rho^{\mathbf{y}} J(\Omega \times \mathbf{u}^{\mathbf{y}})_j$ is a source function representing the potential and Coriolis forces. If ρ is constant, the potential force term can be incorporated into T_{jl} as an additional term, $-\rho^{\mathbf{y}}\Phi^{\mathbf{y}} K_{jl}$.

GLM Formulation and Radiation Stress

Of the various 'curvilinear' coordinate representations, the GLM formulation, in which the mean current velocity ($\overline{\mathbf{u}}^L$) is equal to the mean velocity of fluid particles, is perhaps the approach which is the most dynamically satisfactory. Related equations for the evolution and propagation of waves, in terms of wave action conservation, have also been formulated [3]. If, as is the case for breaking waves and in atmospheric flow over waves, there exist critical layers where the mean flow velocity is equal to the wave phase speed, the amplitude of the oscillatory part of the coordinate transformation tends to infinity and the GLM method breaks down, so a more general coordinate formulation becomes necessary.

The GLM coordinate system is not completely surface-following: the air-water interface has a mean $O[(ak)^2]$ vertical coordinate displacement $\overline{\zeta}^L$ which will vary in space and time [9, 10].

The GLM wave action equation, neglecting dissipative forces, may be written as [3]

$$(J\rho A^{\eta})_{,t} + \nabla_{\mathbf{y}}\cdot(\overline{\mathbf{u}}^L J\rho A^{\eta} + \mathbf{B}^{\eta}) = 0. \tag{3}$$

A^{η} is the *wave action density* if $\eta = -\theta = -(\mathbf{k}\cdot\mathbf{y} - \omega t)$, the wave *energy* density if $\eta = t$, or the wave *pseudomomentum* per unit mass if $\eta = -\mathbf{y}$. To $O[(ak)^2]$, neglecting advection by the mean current,

$$\int_{-h}^{0} \mathbf{B}^{\eta}(y_3)\,dy_3 = \rho\mathbf{C}_g \int_{-h}^{0} A^{\eta}(y_3)\,dy_3. \tag{4}$$

The waves transport wave energy and pseudomomentum, as well as wave action, at the same speed as the wave action.

The quantity $\mathbf{B}^{-\mathbf{x}}$ can be regarded as a radiation stress. When integrated vertically it becomes

$$R_{11} = \int_{-h}^{0} \mathbf{B}_1^{-\mathbf{x}}(y_3)\,dy_3 = E\left(\frac{1}{2} + \frac{kh}{\sinh 2kh}\right) = E\frac{\mathbf{C}_g}{C}. \tag{5}$$

For finite water depth, this is different from the radiation stress computed in a fixed, Eulerian coordinate system [18]:

$$S_{11} = \overline{\int_{-h}^{\zeta}(P + \rho u_1^2)\,dx_3} - \int_{-h}^{\overline{\zeta}} \rho g(\overline{\zeta} - x_3)\,dx_3 = E\left(\frac{1}{2} + \frac{2kh}{\sinh 2kh}\right). \tag{6}$$

However, the result (6) is also obtained in the GLM coordinate system (ξ_j being the coordinate displacement and $(\cdot)^l$ representing wave-associated fluctuations):

$$S_{11} = \overline{\int_{-h}^{0}(P^l + (u_1^l)^2)(1 + \xi_{3,3})\,dy_3} = \int_{-h}^{0}\left(\overline{P^l\xi_{3,3}} + \overline{(u_1^l)^2}\right)dy_3. \tag{7}$$

The difference between the momentum flux S_{11} and the pseudomomentum flux R_{11} is accounted for by an $O[(ak)^2]$ change in the (Eulerian) mean surface elevation [18]:

$$\overline{\zeta} = -\frac{1}{2}\frac{a^2 k}{\sinh 2kh}, \tag{8}$$

which corresponds to the mean Bernoulli pressure reduction at the sea bottom due to the oscillatory wave motion, $\rho\overline{[u_1^l(-h)]^2}$. Horizontal depth variations thus induce mean hydrostatic pressure gradients which account for the variations in $S_{11} - R_{11}$. The GLM surface elevation will become $\overline{\zeta}^L = -\frac{1}{2}a^2k/(\sinh 2kh) + \frac{1}{2}a^2k/(\tanh kh)$, the extra term $(\frac{1}{2}a^2k/\tanh kh)$ being required as a result of mass conservation.

Wave Dissipation and Coupling of Wave and Current Models

The flux of momentum into the wave field from wind forcing, and the flux of momentum from the wave field into the current when waves dissipate, must be taken into account.

If the (eddy) viscosity ν is constant, the wave motion will be irrotational except in a thin vorticity layer near the surface, the wave amplitude will tend to decay with time [16] as $\exp\left(-2\nu k^2 t\right)$, and the wave momentum will be transferred into the water column with an apparent source at the surface.

If we regard the waves as being damped by an eddy viscosity which varies with depth, the rotational component of the wave motion extends to greater depths [12], and the waves tend to decay according to

$$a \propto \exp\left[-2k^2\left(\int_{-\infty}^{0} 2k\nu(y_3)\,e^{2ky_3}\,dy_3\right)t\right].$$

The momentum is then transferred from waves to the current partly at the surface, at a rate given by the surface value of ν, and the rest from a diffuse source distributed within the water column as $\nu_{,3}e^{2ky_3}$.

One can simulate wave dissipation by e.g. wave breaking and white-capping (e.g. [11]), by employing a vertically-varying eddy viscosity which has the same wave-frequency-dependent wave-damping effect. It is impossible to use the same eddy viscosity to damp the wave energy as one uses for the diffusion of momentum within the current field: the former must be much smaller than the latter.

Effect of Surface Films and Sea Ice

The boundary condition at the water surface is changed substantially if a surfactant film, with a surface tension which varies with the extension and compression of surface, is present. The rate of viscous damping of surface waves is increased dramatically, from $2\nu k^2$ to a value of the order $k(\nu\omega)^{1/2}$ [8, 19], and this more rapid conversion of wave momentum to mean flow leads to there being a stronger wave-induced current in the surface viscosity layer [27].

A similar effect also occurs if there is a thin layer of viscous (or viscoelastic) fluid at the surface, for example a layer of oil or thin ice [15]. In the marginal ice zone, the change in the near-surface wave-induced current between ice-covered and open-water areas may cause upwelling in the vicinity of the ice edge [26].

Concluding remarks

We have given here necessarily a very brief description of some effects which should be taken into account in the coupled modelling of waves and currents, particularly near the sea surface. Notwithstanding the difficulties which may arise in the analysis, the use of surface-following coordinates has its advantages, since such coordinates enable a fine resolution of what are expected to be large gradients in the dependent variables in the cross-interface direction. This is particularly important if we also wish to consider thermal effects and the mass flux through the air–sea interface, since such large gradients are indeed observed [7, 21, 22, 24, 25].

It should also be noted that a self-consistent formulation of the wave–current interaction problem will include the vortex force [6, 20] which provides a mechanism for the generation and maintenance of Langmuir circulations. An additional case which may be described within this coupled framework is the 'remote recoil' effect described by Bühler and McIntyre [4].

Acknowledgement

This work was supported by the Aurora Mobility Programme for Research Collaboration between France and Norway, funded by the Research Council of Norway (NFR) and The French Ministry of Foreign Affairs / Ministry of Education, Research and Technology. A. D. Jenkins acknowledges support from NFR under project no. 155923/700 (ProClim).

REFERENCES

[1] J. L. Anderson, S. Preiser, and E. L. Rubin. Conservation form of the equations of hydrodynamics in curvilinear coordinates. *Journal of Computational Physics*, 2:279–287, 1968.

[2] D. G. Andrews and M. E. McIntyre. An exact theory of nonlinear waves on a Lagrangian-mean flow. *Journal of Fluid Mechanics*, 89:609–646, 1978.

[3] D. G. Andrews and M. E. McIntyre. On wave-action and its relatives. *Journal of Fluid Mechanics*, 89:647–664, 1978.

[4] O. Bühler and M. E. McIntyre. Remote recoil: a new wave–mean interaction effect. *Journal of Fluid Mechanics*, 492:207–230, 2003.

[5] M.-S. Chang. Mass transport in deep-water long-crested random gravity waves. *Journal of Geophysical Research*, 74:1515–1536, 1969.

[6] A. D. D. Craik and S. Leibovich. A rational model for Langmuir circulations. *Journal of Fluid Mechanics*, 73:401–426, 1976.

[7] C. J. Donlon and I. S. Robinson. Observations of the oceanic thermal skin in the Atlantic Ocean. *Journal of Geophysical Research*, 102(C8):18 585—18 606, 1997.

[8] R. Dorrestein. General linearized theory of the effect of surface films on water ripples, I–II. *Proceedings of the Koninklijke Nederlandse Akademie van Wetenschappen, Series B*, 54:260–272 & 350–356, 1951.

[9] R. Grimshaw. Wave action and wave–mean flow interaction, with application to stratified shear flows. *Annual Review of Fluid Mechanics*, 16:11–44, 1984.

[10] J. Groeneweg and G. Klopman. Changes of the mean velocity profiles in the combined wave–current motion described in a GLM formulation. *Journal of Fluid Mechanics*, 370:271–296, 1998.

[11] K. Hasselmann. On the spectral dissipation of ocean waves due to white capping. *Boundary-Layer Meteorology*, 6:107–127, 1974.

[12] A. D. Jenkins. Wind and wave induced currents in a rotating sea with depth-varying eddy viscosity. *Journal of Physical Oceanography*, 17(7):938–951, 1987.

[13] A. D. Jenkins. Conservation form of the momentum equation in a general curvilinear coordinate system. *Ocean Modelling (newsletter)*, 84:6–8, 1989. Unpublished manuscript, available from the Robert Hooke Institute, Dept. of Atmospheric, Oceanic and Planetary Physics, Clarendon Laboratory, Parks Road, Oxford, OX1 3PU, U.K. Manuscript available for download at URL http://www.gfi.uib.no/˜ jenkins/papers/JenkinsAD_OM-1989-6.ps.gz.

[14] A. D. Jenkins. A quasi-linear eddy-viscosity model for the flux of energy and momentum to wind waves, using conservation-law equations in a curvilinear coordinate system. *Journal of Physical Oceanography*, 22(8):843–858, 1992.

[15] A. D. Jenkins and S. J. Jacobs. Wave damping by a thin layer of viscous fluid. *Physics of Fluids*, 9(5):1256–1264, 1997.

[16] H. Lamb. *Hydrodynamics*. Cambridge University Press, Cambridge, England, 6th edition, 1932.

[17] M. S. Longuet-Higgins. Mass transport in water waves. *Philosophical Transactions of the Royal Society of London, Series A*, 245:535–581, 1953.

[18] M. S. Longuet-Higgins and R. W. Stewart. Radiation stress and mass transport in gravity waves with application to 'surf beats'. *Journal of Fluid Mechanics*, 13:481–504, 1962.

[19] C. Marangoni. Sul principio della viscosità superficiale dei liquidi stabili. *Nuovo Cimento, Series 2*, 5/6:239–273, 1872.

[20] J. C. McWilliams and J. M. Restrepo. The wave-driven ocean circulation. *Journal of Physical Oceanography*, 29:2523–2540, 1999.

[21] L. Memery and L. Merlivat. Modelling of gas flux through bubbles at the air–water interface. *Tellus*, 37B:272–285, 1985.

[22] W. L. Peirson and M. L. Banner. Aqueous surface layer flows induced by microscale breaking wind waves. *Journal of Fluid Mechanics*, 479:1–38, 2003. DOI: 10.1017/S0022112002003336.

[23] W. J. Pierson. Perturbation analysis of the Navier–Stokes equations in Lagrangian form with selected linear solutions. *Journal of Geophysical Research*, 67:3151–3160, 1962.

[24] P. Schlüssel, W. J. Emery, H. Grassl, and T. Mammen. On the bulk skin temperature difference and its impact on satellite remote sensing of sea surface temperature. *Journal of Geophysical Research*, 95:13 341—13 356, 1990.

[25] A. V. Soloviev and P. Schlüssel. Parameterization of the cool skin of the ocean and of the air–ocean gas transfer on the basis of modelling surface renewal. *Journal of Physical Oceanography*, 24:1339, 1994.

[26] J. E. Weber. Wave attenuation and wave drift in the marginal ice zone. *Journal of Physical Oceanography*, 17:2351–2361, 1987.

[27] J. E. Weber and E. Førland. Effect of an insoluble surface film on the drift velocity of capillary–gravity waves. *Journal of Physical Oceanography*, 19:952–961, 1989.

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