

Reply

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Eifler 2005 criticises our model (Jenkins and Ward 2005, referred to as *JW*) for a number of reasons:

- (1) If our model is run with a step-function wind and wind stress applied at time $t = 0$, the velocity field $u(z, t)$ becomes

$$u(z, t) = \lambda U \exp\left(-\frac{\lambda U z}{u^{*2} t}\right), \quad (1)$$

and satisfies

$$\frac{\partial u}{\partial t} = \frac{u^{*2} z}{\lambda U} \frac{\partial^2 u}{\partial z^2}, \quad (2)$$

where z is the downward-pointing vertical coordinate, λU is the interface velocity, U is the wind speed, and u^* is the waterside friction velocity. In

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JW we state that the quantity

$$\varepsilon_{app} = \frac{u^{*2}z}{\lambda U} \quad (3)$$

plays a role similar to that of an eddy viscosity. *Eifler* disagrees with our interpretation, using a stricter definition of eddy viscosity ε , in our one-dimensional case it is defined using the equation

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial z} \left(\varepsilon \frac{\partial u}{\partial z} \right), \quad (4)$$

from which he deduces from (2) that

$$\varepsilon = \frac{u^{*2}}{\lambda U} z + \left(\frac{u^{*2}}{\lambda U} \right)^2 t, \quad (5)$$

so that the vertical momentum flux ‘has to be sustained by an eddy viscosity tending towards an infinite value’.

- (2) *Eifler* computes the time dependence of the mixed-layer depth (defined for the *JW* model as the depth where the local vertical momentum flux is a specified fraction of the surface momentum flux), and compares it with an ‘empirically-tested standard formulation’. The mixed-layer depth z_{ml} of the *JW* model obeys

$$z_{ml} = \frac{A^{T=x\%}}{16} u^* t, \quad (6)$$

where $A^{T=x\%}$ is defined by *Eifler*’s Eq. 10. The empirically-tested standard formulation gives a mixed-layer depth D_{ml} which obeys

$$D_{ml} = 10.5 u^* t^{1/2}, \quad (7)$$

the units of D_{ml} , u^* , and t being metres, m s^{-1} , and seconds, respectively. The different time dependence of the mixed layer depths in Eqs. 6 and 7 is stated to be a reason against the validity of the *JW* model.

(3) The derived numerical thermal model of *JW* shows in Fig. 3 (of *JW*) a mixed layer depth which develops linearly in time, and the agreement with observations is not particularly good in this case, giving further evidence against the usefulness of the *JW* model.

Although *Eifler* raises genuine issues which question the validity and applicability of the *JW* model, we are nevertheless of the opinion that it has a degree of usefulness: indeed, *Eifler* does state in his conclusion that *JW* ‘have provided a promising analytical solution of the vertical transport equation . . . with very interesting mathematical properties’.

Regarding *Eifler*’s Reason 3 above, we concede that the model (not surprisingly, given its simplicity) does not always give good agreement with the measured data, although the particular reasons for this may also include horizontal advection effects. Of course, perfect dynamical agreement cannot in any case be obtained, since the *JW* model assumes zero Coriolis force, neutral stratification, and infinite water depth.

Reason 1, that the eddy viscosity ε tends to infinity for infinite time under a steady applied wind stress, is not such a serious objection as it first appears. Equation 5 may be re-written as

$$\varepsilon = \frac{u^{*2}}{\lambda U} \left[z + \left(\frac{u^{*2}}{\lambda U} \right) t \right], \quad (8)$$

from which we see that the graph of ε versus z shifts upwards at a velocity $[u^{*2}/(\lambda U)]$. This result may also be interpreted as an increase in the length scale of the dominant turbulent eddies which impact the surface (and other depths). The increase of ε to infinity at infinite time is a consequence of the depth being infinite. The fact that the near-surface values of the eddy viscosity

are no longer close to linearly proportional to depth is in fact reminiscent of the breaking-wave enhanced values of near-surface eddy viscosity shown in Figs. 3.18, 3.19, and 7.16 of Burchard (2002).

Reason 2, that the JW linear t dependence of mixed layer depth z_{ml} is in disagreement with the empirically-tested $t^{1/2}$ dependence of mixed layer depth D_{ml} , is rather interesting. Equation 7 corresponds to Eq. 6.1 in Burchard (2002), which was suggested by Price (1979). The constant 10.5 in (7) is in fact $1.05N_0^{-1/2}$, with N_0 , the initial Brunt-Väisälä frequency, being set to 10^{-2} s^{-1} . For an initially linearly-stratified water column, or, in fact, for a water column with any stably-stratified density profile, subjected to a constant input of turbulent energy at the surface for mixing, the result $D_{ml} \propto t^{1/2}$ is a consequence of the requirement that the potential energy of the water column increases at a constant rate due to mixing. Similarly, the result $z_{ml} \propto t$ is a consequence of a constant rate of momentum input to the water column. If the Coriolis force due to the Earth's rotation is taken into account, z_{ml} can be expected not to increase indefinitely, as the wind-induced current would be restricted to a boundary layer of Ekman depth typically of the order of magnitude of u^*/f (Madsen 1977). Further study is required to resolve these different boundary layer time dependences, and to evaluate the joint effects of stratification and rotation within the simple JW model framework.

References

- H. Burchard. *Applied Turbulence Modelling in Marine Waters*. Springer, Berlin, 2002, 229 pp.
- W. Eifler. Comment on the Paper "A Simple Model for the Short-Time Evo-

- lution of Near-Surface Current and Temperature Profiles” (A. D. Jenkins and B. Ward). *Deep-Sea Res.*, 000:000–000, 2005.
- A. D. Jenkins and B. Ward. A simple model for the short-time evolution of near-surface current and temperature profiles. *Deep-Sea Res.*, 000:000–000, 2005.
- O. S. Madsen. A realistic model of the wind-induced Ekman boundary layer. *J. Phys. Oceanogr.*, 7:248–255, 1977.
- J. F. Price. On the scaling of stress-driven entrainment experiments. *J. Fluid Mech.*, 90:509–529, 1979.