Wave—mean flow interaction in coupled atmosphere–ice–ocean systems

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'Mission Statement'

A dynamically consistent framework for modelling atmosphere–ocean interaction must take account of surface waves, either implicitly or explicitly. We may account for the waves explicitly by employing a numerical spectral wave model, and applying a suitable theory of wave—mean flow interaction.

Introduction

- Surface waves comprise an important aspect of the interaction of the atmosphere and the ocean.
- A *dynamically consistent framework* for the modelling of atmosphere–ocean interaction *must take surface waves into account*:
  - — *implicitly*, via a Charnock-type relation between air–sea momentum flux and wind speed, or
  - — *explicitly*, using, e.g., a spectral wave prediction model and analytical or numerical models for the interaction between the wave field in the atmospheric and oceanic surface boundary layers.
- Wave—mean flow interaction may be by various mechanisms, including:
  - — Wave generation by wind (e.g. Miles [16], Janssen [5], Jenkins [7])
  - — Flux of momentum from waves to current, e.g. by *breaking* [19], *turbulent stresses* within the water column [6, 10] or at the sea bottom [14]
  - — *inertial coupling theory* [2, 3] — wave field acts to transport momentum vertically, from the critical level in the atmospheric boundary layer, where the dominant wave phase speed = wind speed, to a level below the wave troughs where the current ≈ wave-induced Stokes drift.

Coordinate systems

There are advantages to using curvilinear coordinate systems which follow the water surface:

- We may resolve vertical variations at small distances from the surface
  - — can be essential for applications such as *heat and gas exchange* through the water surface, and *ice* formation.
- *Time-independent curvilinear coordinates* [e.g. 13]: Can be used with surface waves of fixed form.
- *Lagrangian coordinates* [e.g. 4, 9, 6, 17]: Coordinate transformation may distort unacceptably for long times (t).
- *Generalized Lagrangian mean* (GLM) formulation [1]: Not quite surface following, coordinate transformation becomes singular at critical levels.
- *'Sigma'-coordinates:* Vertical coordinate displacements only [e.g. 15] — may be insufficiently general.
- *General, time-dependent* curvilinear coordinates [e.g. 7]: Includes Eulerian (Cartesian) and the other coordinate systems mentioned above, as special cases

General, time-dependent coordinates

Hydrodynamic equations in Cartesian coordinate system $\mathbf{x} = (x_1, x_2, x_3)$:

$$\rho^{\mathbf{x}}\left[u^{\mathbf{x}}_{j,t} + u^{\mathbf{x}}_l u^{\mathbf{x}}_{j,l} + \Phi^{\mathbf{x}}_{,j} + 2(\Omega \times \mathbf{u}^{\mathbf{x}})_j\right] - \tau^{\mathbf{x}}_{jl,l} = 0,$$
$$\rho^{\mathbf{x}}_{,t} + u^{\mathbf{x}}_l \rho^{\mathbf{x}}_{,l} + \rho^{\mathbf{x}} u^{\mathbf{x}}_{l,l} = 0, \qquad (1)$$

where ρ is the fluid density, $\mathbf{u} = (u_1, u_2, u_3)$ is the velocity, Ω is the rotational angular velocity vector, Φ is a force (e.g. gravitational) potential and the tensor τ_{jl} incorporates both pressure and shear stress. Repeated indices are summed from 1 to 3.

In curvilinear coordinates $\mathbf{y} = (y_1, y_2, y_3)$ with Jacobian $J = \det[x^{\mathbf{y}}_{j,l}]$, cofactors K_{jl}, equation 1 becomes

$$P_{j,t} - T_{jl,l} = S_j, \quad (\rho^{\mathbf{y}} J)_{,t} + [K_{ml}\rho^{\mathbf{y}}(u^{\mathbf{y}}_m - x^{\mathbf{y}}_{m,t})]_{,l} = 0, \qquad (2)$$

where $P_j = \rho^{\mathbf{y}} J u^{\mathbf{y}}_j$ is the 'concentration of x_j-momentum in $\mathbf{y}$-space', $T_{jl} = \left[\tau^{\mathbf{y}}_{jm} - \rho^{\mathbf{y}} u^{\mathbf{y}}_j \left(u^{\mathbf{y}}_m - x^{\mathbf{y}}_{m,t}\right)\right] K_{ml}$ is minus the flux of x_j-momentum across y_l-surfaces, and $S_j = -\rho^{\mathbf{y}}\Phi^{\mathbf{y}}_{,l} K_{jl} - 2\rho^{\mathbf{y}} J (\Omega \times \mathbf{u}^{\mathbf{y}})_j$ is a source function representing the potential and Coriolis forces.

If ρ is constant, the potential force term can be incorporated into T_{jl} as an additional term, $-\rho^{\mathbf{y}}\Phi^{\mathbf{y}} K_{jl}$.

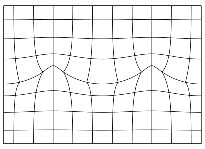

Figure 1. Example of a coordinate system, above and below a wave surface. In this particular case we have $\mathbf{x} = \left(y_1 - ae^{-k|y_3|}\sin(ky_1 - \omega t),\ y_2,\ ae^{-k|y_3|}\cos(ky_1 - \omega t)\right)$. Above the interface, the coordinate system is isomorphic: below the interface $J = 1 + O\left(\varepsilon^2\right)$.

Perturbation expansion

- In water column (wave-induced current)
- Coordinate transformation $\mathbf{x}^{\mathbf{y}} = \mathbf{y} + \xi$
- Quasilinear, wave slope ε, expansion of any variable ϕ:

$$\phi \approx \overline{\phi^{(0)}} + \Re \sum_{\mathbf{k}} \phi^{(1)}_{\mathbf{k}} \exp[k_j y_j - \omega(\mathbf{k})t] + \overline{\phi^{(2)}}, \qquad (3)$$

  Neglect $O[(\Omega/\omega)^n]$ terms for $n \geq 2$.
- Wave dissipation by an assumed eddy viscosity ν, which may be complex and frequency-dependent, but independent of y_1, y_2, and t
- Finite water depth

Summary of results to $O(\varepsilon)$:

- For coordinate transformation given by $\xi^{(1)}_{j,t} = \alpha_j u^{(1)}_j$:
- Eliminating pressure terms, the $O(\varepsilon)$ equations are *independent of the coordinate system chosen*
- Predominantly potential flow in interior of fluid, with a small rotational component determined by $\partial\nu/\partial y_3$
- Vorticity layers near surface/under ice [8, 12, 18] and near bottom [14]
- Wave dissipation rate determined by a weighted integral of ν with respect to y_3

Results for mean flow $[O(1) + \overline{O(\varepsilon^2)}]$

- 'Natural' momentum equation for 'quasi-Eulerian' mean current [1, 9]. Coriolis force acts on Lagrangian mean current including Stokes drift
- Momentum transferred from atmosphere to waves during wind-wave generation
- Momentum transferred from waves to current:
  - — in surface/under-ice/bottom boundary layers
  - — in water column, at a rate dependent on $\partial\nu/\partial y_3$
- Figure 2 shows early results using a single-point wave model (WAM) [10]

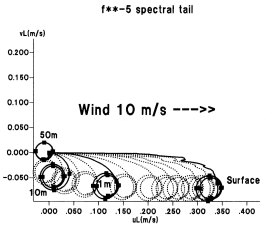

Figure 2. (After Jenkins [10].) Evolution of the near-surface drift current when the wave field is calculated using a 1-point version of the WAM wave model [11]). The wave height increases to 1.8 m after 36 hours. The oscillations in the current during the first few hours may be due to the wave model numerics. (Reprinted from *Dt. Hydrogr. Z.*, © 1989 BSH.)

Work in progress and future plans:

- Further analysis to $O(\varepsilon^2)$ of the coupled system in general curvilinear coordinates, for finite water depth.
- Application of the results to coupling numerical models for the atmosphere, the wave field, and the water column, including the effects of sea ice, surface films, and Langmuir circulations. Investigation of air–sea fluxes of heat and mass (gas species, aerosols).

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