Statistical mechanics and our society's future

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Introduction

- In recent years, some scientists have applied the principle of *maximum entropy production* (MEP) to various non-equilibrium physical and biological systems:
  - turbulent boundary layers
  - the ‘sand pile’ system
  - the atmospheric circulation of the Earth and other planets
  - the ocean meridional overturning circulation
  - ecosystems
- The theoretical basis of MEP has been strengthened by *Roderick Dewar*’s proof, using principles from information theory, that trajectories in phase space have probabilities which *increase rapidly as their entropy production increases*.

- In my opinion, the principle of maximum entropy production may also be applied to human economic systems.
- This may be illustrated by historical examples
- There are implications for the future course of economic and social development, given the current abundance of fossil fuel energy and raw materials.
- Manuscript under review at *Ecological Economics*, may be downloaded from `http://www.arXiv.org/cond-mat/0503308`

- The term *entropy* was introduced by Rudolph Clausius in 1865 as a function of the thermodynamic state of a system:

$$S_B - S_A \geq \int_A^B dQ/T,$$ with equality only if $A \to B$ is reversible

- *Ludwig Boltzmann*[a] determined the phase volume W of an ideal gas (N atoms, energy between E and $E + dE$, integrate for x_i in volume V and momentum p_i satisfying $E < \sum p^2/(2m) < E + dE$:

$$W = \int_R d^3x_1 \dots d^3x_N d^3p_1 \dots d^3p_N = CV^N E^{3N/2-1} dE,$$

from which he obtained $S = k \log W$.

[a] see E. T. Jaynes 'The Evolution of Carnot's Principle', EMBO workshop 1984

- J. Willard Gibbs postulated that a system would tend to its state of maximum entropy, *subject to constraints* given by total energy, composition, etc., since the state of maximum entropy is *overwhelmingly the most probable*.
- *Boltzmann*'s phase volume indicates the probability of the state. (Quantum mechanics discretizes the phase volume in units of $\hbar^{3N}$, but is not necessary in order to define entropy.)
- *Claude Shannon* introduced the concept of entropy in information theory, giving rise to maximum entropy methods in statistical estimation and data analysis.

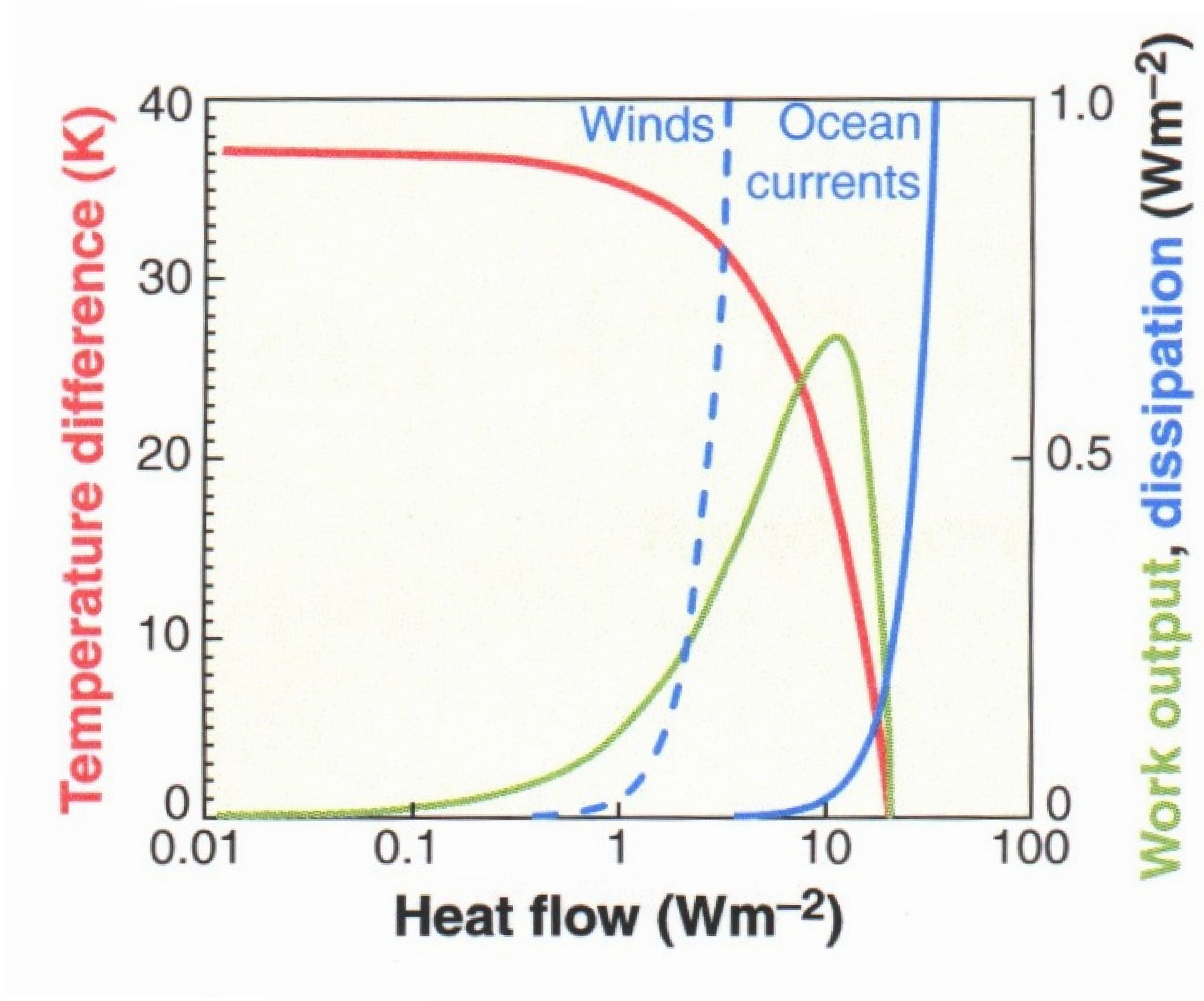
Non-Equilibrium Systems

- A mathematical theory of a non-equilibrium phenomenon—that of heat conduction—was developed by *Fourier* in 1826.
- The explanation of such non-equilibrium processes in terms of the kinetic theory of gases was developed by Maxwell, Boltzmann, Enskog, and Chapman, in the period $\approx$1870–1920.
- A general quantitative theory of non-equilibrium thermodynamics was postulated by *Lars Onsager* (1931), valid for systems close to thermodynamic equilibrium.

- For systems far from thermodynamic equilibrium, the general statistical mechanical theory has been incomplete.
- *E. T. Jaynes* made a detailed systematic study of the maximum entropy principle for physical systems, and was of the opinion that for non-reversible processes, the rate of production of entropy should be a maximum, again *subject to appropriate constraints*.

Maximum entropy production (MEP)

- Applied to:
  - fluid turbulence from 1950s (F. H. Busse, W. V. R. Malkus),
  - the global climate from 1970s (G. Paltridge),
  - the ocean thermohaline circulation (Shimokawa & Ozawa),
  - the circulation of various planets (R. Lorenz et al.)



From R. Lorenz, *Science*, 2003 February 7: Results from a two-box model of the Earth’s climate.

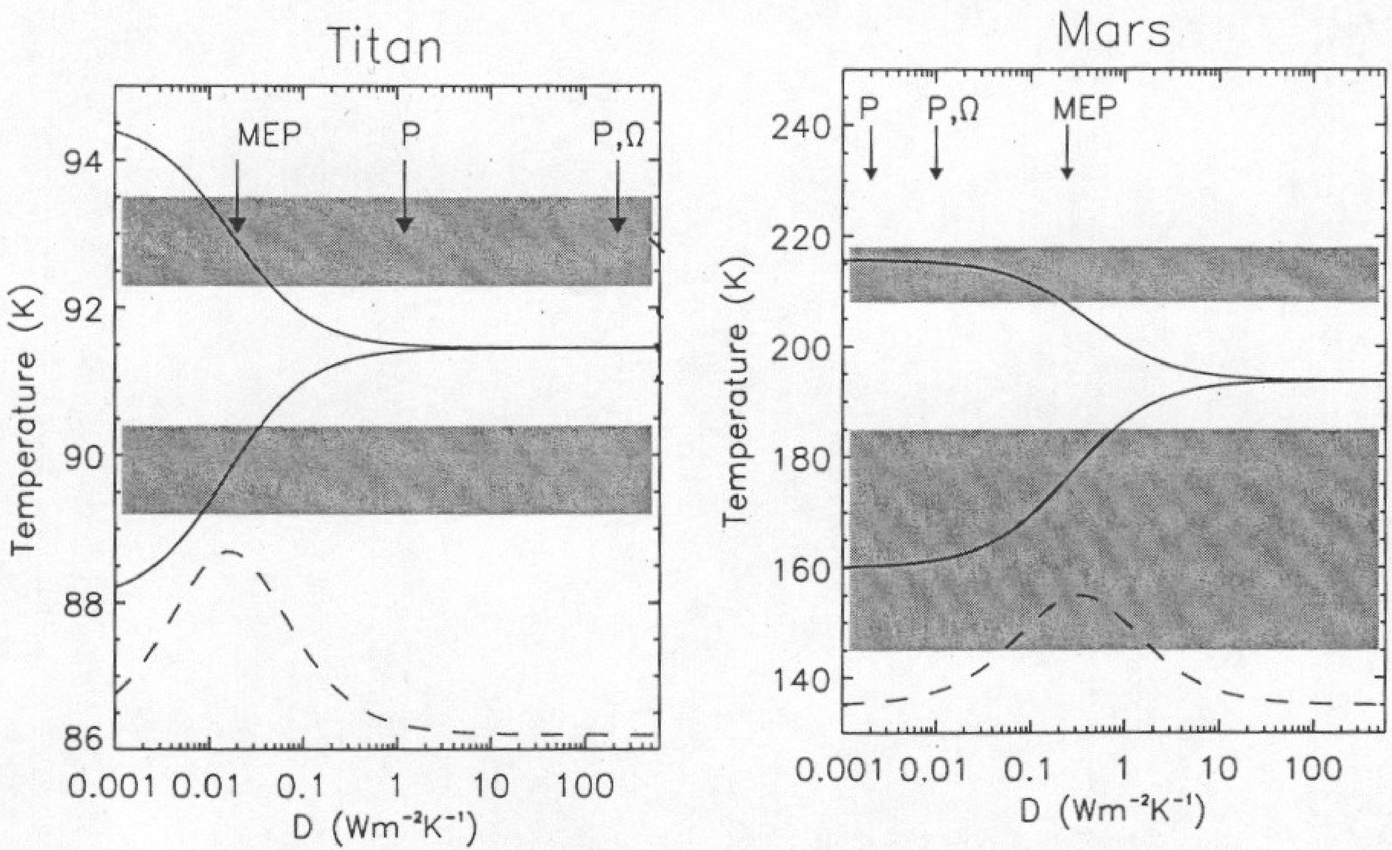


Figure 2. Observed temperature contrasts on Titan (a) and Mars (b) agree with predictions at maximum entropy production. Shading indicates observed annually-averaged temperature ranges for the regions 10- 20 deg and 40-60 deg latitude. Solid curves are model temperatures (upper curve tropics; lower curve polar regions) as a function of D : dashed curves at bottom are the entropy production $dS/dt=(F/T_1-F/T_0)$ in arbitrary units. It is seen that the temperatures are successfully predicted where dS/dt has a maximum. Values of D predicted from scaling pressure (P), or pressure and rotation rate (P,Ω) are marked: the arrows marked MEP correspond to the maximum entropy value inferred from the simple expression $D=\sigma T^3/2$. High ($>10 Wm^{-2}K^{-1}$) values for D predict near-isothermal conditions and do not agree with observations. Low ($<0.01 Wm^{-2}K^{-1}$) D values appear compatible for Mars but not for Titan, but see text for discussion of the failure of small D for the Mars case. Note that the temperature slopes are steepest at MEP.

(From R. Lorenz et al., Geoph.R.Letts 28:415, 2001.) Max. entropy production explains the circulation of the 3 bodies better than if one assumes that meridional heat transport is proportional to atmospheric pressure.

Garth Paltridge used the maximum entropy production principle in studies of the Earth's climate in papers from 1975 onward. His work forms the basis of more recent studies by H. Ozawa, S. Shimokawa, and colleagues.

HISASHI OZAWA, SHINYA SHIMOKAWA, AND HIROFUMI SAKUMA PHYSICAL REVIEW E 64 026303

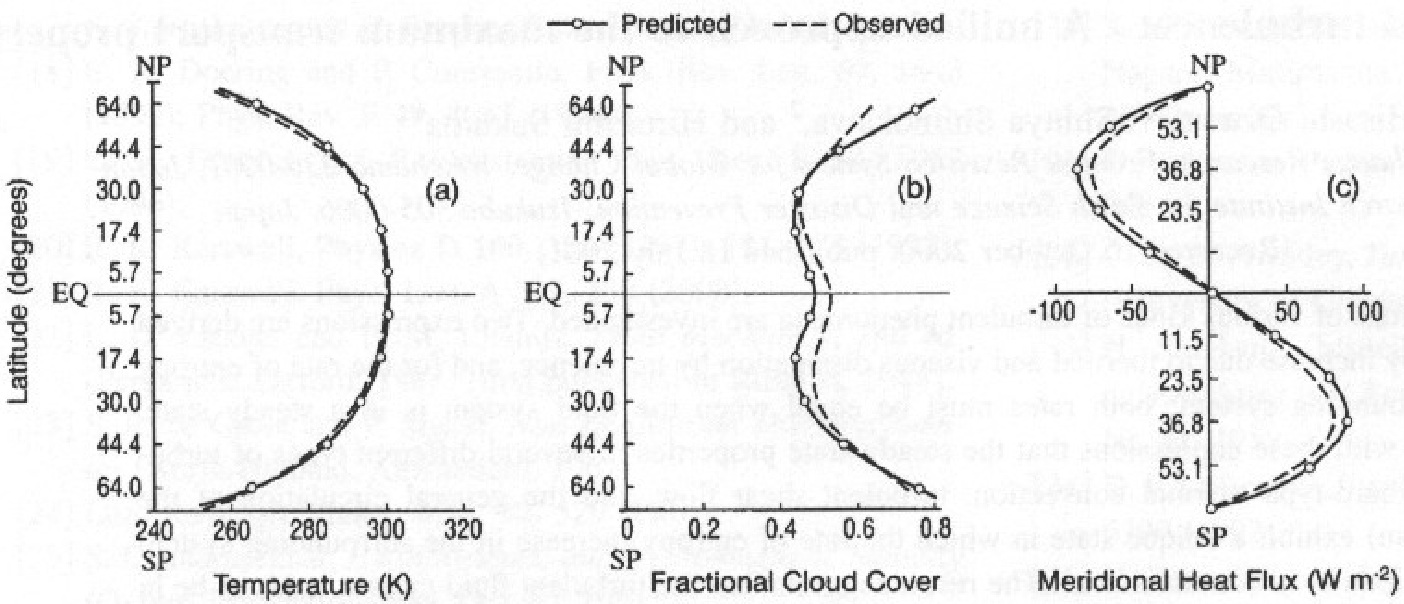


FIG. 1. Global distributions of: (a) mean air temperature, (b) cloud cover, and (c) horizontal heat transport in the earth. Solid line: predicted with $\dot{S}_{\mathrm{turb,st}} = \mathrm{Max.}$ and dashed line: observed (after Paltridge [10]).

Dewar's proof

- 'Information theory explanation of the fluctuation theorem, maximum entropy production and self-organized criticality in non-equilibrium stationary states' [J. Phys. A 36:631 (2003)].
- Uses *Gibbs'* formulation of ensemble statistical mechanics, which maximises $-\sum_i p_i \log p_i$ with respect to the microstate probabilities p_i, but *instead* we consider probabilities p_Γ *of the paths* Γ in the phase space.
- We maximise subject to constraints on the initial energy and mass density (of various constituents) $d(x, 0)$, and the flux $F_n(x)$ of energy and mass at the boundaries of the system.

- We obtain a probability distribution of paths:

$$p_\Gamma = \frac{1}{Z} \exp A_\Gamma, \qquad Z = \sum_\Gamma \exp A_\Gamma,$$

$$A_\Gamma = \int_V \lambda(x) \cdot d(x,0)_\Gamma + \int_\Omega \eta(x) \cdot F^n(x)_\Gamma$$

- A_Γ is the *path action*, Z is the *partition function*, $\lambda(x)$ and $\eta(x)$ are *Lagrange multipliers*

After considering the conditions at the boundaries, integrating by parts etc., we have

$$A_\Gamma(\lambda) = \frac{1}{2}\int_V \lambda \cdot (d_{\Gamma(0)} + d_{\Gamma(\tau)}) - \frac{\tau}{2}\int_V (F_\Gamma \cdot \nabla\lambda + \lambda \cdot Q_\Gamma)$$

(Q is the source term for d, and τ is the length of the time interval)

In terms of more usual thermodynamical variables: temperature T, chemical potentials μ_i, internal energy u, concentrations ρ_i, *rate of entropy production* σ:

$$\lambda = \frac{1}{kT}(-1, \{\mu_i\}), \qquad H = u - \sum_i \mu_i \rho_i$$

$$A_\Gamma(T, \mu_i) = -\frac{1}{2}\int_V \frac{H_{\Gamma(0)} + H_{\Gamma(\tau)}}{kT} + \frac{\tau\sigma_\Gamma}{2k}$$

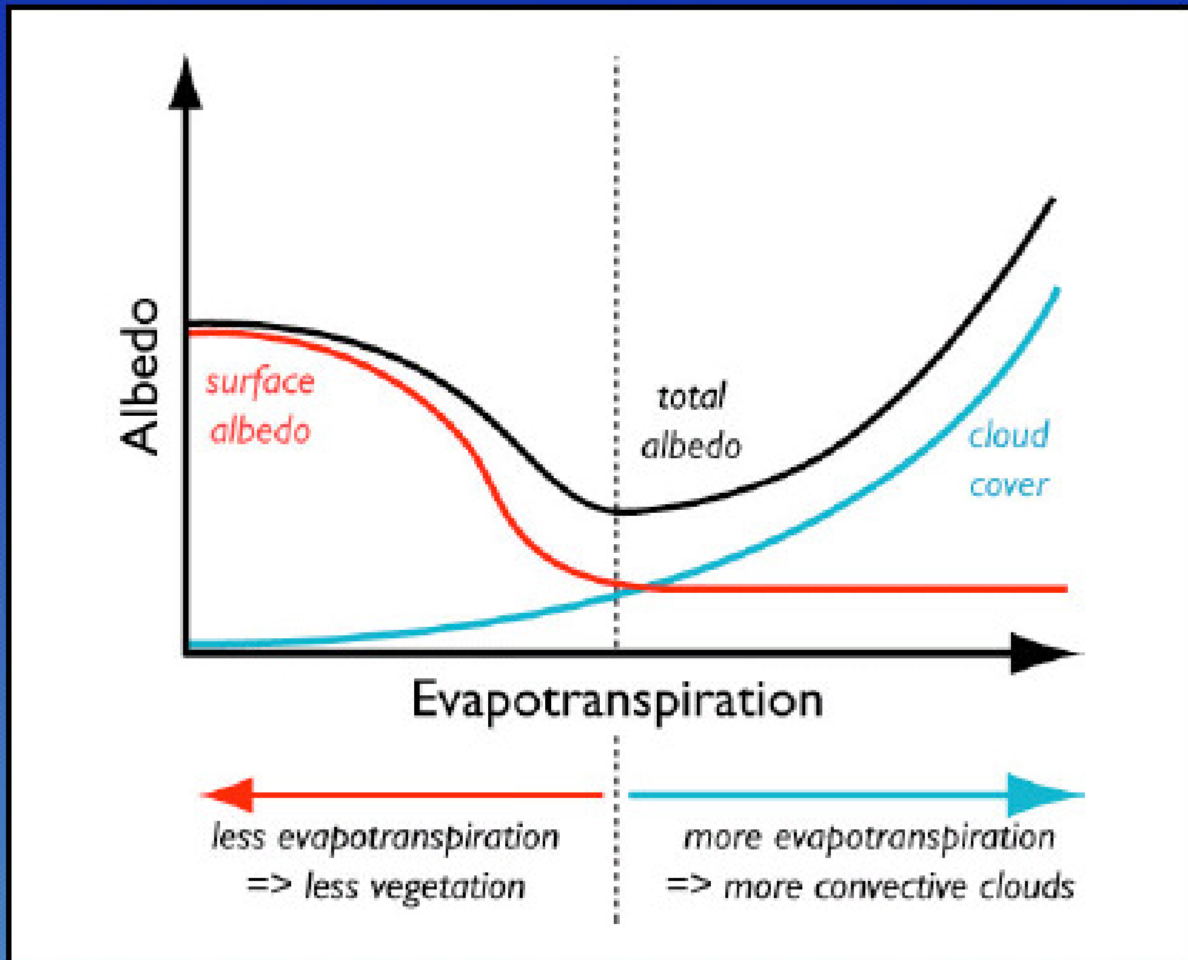
So if the system is in a steady state, the entropy production rate should be maximised, since the probability of path Γ will be proportional to $\exp(\tau\sigma_\Gamma/(2k))$.

Dewar also considers *self-organized criticality* in systems such as *sand piles*.

Biological and ecological processes

- Living systems tend to have a high degree of order (negative entropy), but they maintain this state by ‘feeding’ on energy and mass sources with a low specific entropy, and producing waste products with a higher entropy.
- Vegetation on the planetary surface tends to reduce both the temperature and the albedo of the surface, thus increasing the entropy production rate for a given input of solar radiation (e.g. Ulanowicz & Hannon 1987; Schneider & Kay 1994; A. Kleidon et al. 2000; Kleidon & Lorenz 2004)

Terrestrial Vegetation and MEP



Kleidon and Fraedrich (submitted)

Human Activity

- One may consider human activity as an extension of non-equilibrium chemical, biological, and ecological processes
- As such, it may be subject to the same considerations (phase-space volume, ‘degrees of freedom’, etc., as physical and biological processes
- Entropy production by/for humans was increased by:
  - the use of *fire* to improve the availability of game etc.
  - the application of *agriculture*, enabling an increase in polulation
  - *smelting of metals*
  - *industrial technology* using *fossil fuels*
  - industrial *nitrogen fixation* (Haber process)
  - etc.

Economic systems

- R. Lorenz (poster, EGU 2003) postulated that max. entropy production may apply to markets and economic systems.
- Self-organized criticality, ‘energy dissipation’ corresponds to profit (difference between buying and selling price).
- Similar considerations may apply to ecological systems.
- However, social science (and biological) applications of the principle may be based on the *usual thermodynamic definitions* of entropy and energy:

- Road transport, being less energy efficient than rail or ship transport, will be *favoured* by the max. entropy production principle.
- The *invasion of Iraq* was likewise more productive of entropy than patient diplomacy.
- The *war on drugs* will be favoured over rational treatment methods.
- . . . *large, international symposia* on climate change produce more entropy than Internet communication.
- To avoid unfortunate consequences of max. entropy production, *thermodynamically realistic constraints* need to be applied.

Additional examples

- The ‘entropy production’ of an agricultural society at a state in which the population is more-or-less constant (P. Laslett 1971) may be maximised by the export of food and excess population to urban areas (which have higher death rates).
- Social stratification may enable a more ‘effective’ entropy production.
- Laslett also states that the controlling *gentry* in 17th century England ‘pressed, like the atmosphere, evenly, over the whole face of England’, that is, as in a body of gas evenly distributed throughout a container, their *spatial distribution* was in a state of maximal entropy.

- The tendency to the reduction in size of families in ‘developed’ societies may be seen as a consequence of the individual families’ strategies for maximization of their entropy production.
- The tendency to *economic growth* corresponds to a maximization of entropy production, subject to constraints
- *Government policy*, what ever it is, will generally act as a *constraint*, and thus reduce entropy production and growth
- *Older, unprofitable industries* will, by their entropy production (employment), lead to (political) pressure to keep them going.

- The *human development index* (HDI) will be less well correlated with entropy production than GNP/GDP. Thus, societies with higher GNP/GDP and relatively lower HDI may be more ‘competitive’.
- The Montreal protocol on ozone protection is easier to implement than entropy production reducing CO_2 emission constraints
- Energy and natural resource availability which is in excess of the ability of a local economy to absorb it, will tend to ‘wasteful’ entropy production e.g. corruption, civil unrest/conflict etc.

Future prospects and challenges

- Environmental protection measures which reduce entropy production / energy consumption will be (politically) difficult to apply
- However, if ‘business as usual’ is seen to lead to a future economic (entropy production) collapse, there may be political pressure to take some necessary ameliorating measures.
- Apparently ‘wasteful’, energy-intensive methods for reducing greenhouse gas emissions, such as compression of CO_2 and injection into depleted petroleum reservoirs, will be favoured by the MEP principle

- If nuclear technology is seen to provide a large energy availability, there will be pressure to (re)introduce it.
- A large excess availability of energy and raw materials may have unfortunate consequences in the form of conflicts to ‘burn them off’.
- New policies put in place will almost invariably, as constraints, lead to reduced entropy production / economic gowth. To avoid ‘loss of competitiveness’, such policies should be implemented by international agreement.